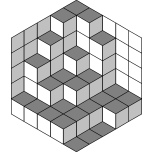




Balkan MO 2008

Macedonia



- 1] Given a scalene acute triangle ABC with $AC > BC$ let F be the foot of the altitude from C . Let P be a point on AB , different from A so that $AF = PF$. Let H, O, M be the orthocenter, circumcenter and midpoint of $[AC]$. Let X be the intersection point of BC and HP . Let Y be the intersection point of OM and FX and let OF intersect AC at Z . Prove that F, M, Y, Z are concyclic.
- 2] Is there a sequence a_1, a_2, \dots of positive reals satisfying simultaneously the following inequalities for all positive integers n : a) $a_1 + a_2 + \dots + a_n \leq n^2$ b) $\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} \leq 2008$?
- 3] Let n be a positive integer. Consider a rectangle $(90n + 1) \times (90n + 5)$ consisting of unit squares. Let S be the set of the vertices of these squares. Prove that the number of distinct lines passing through at least two points of S is divisible by 4.
- 4] Let c be a positive integer. The sequence a_1, a_2, \dots is defined as follows $a_1 = c$, $a_{n+1} = a_n^2 + a_n + c^3$ for all positive integers n . Find all c so that there are integers $k \geq 1$ and $m \geq 2$ so that $a_k^2 + c^3$ is the m th power of some integer.